Suggested Ideas and Approaches for a Transition Year Programme

1. Algebra

✓ <u>**Key Skills: To develop students' fluency and skills in relation to:**</u>

- Factoring expressions
- Common factors
- Difference of two squares
- Sum and difference of two cubes (Higher)
- The perfect square
- Functions

The approach to adopt is that the students begin to recognise patterns in algebraic expressions and are seeking out the different identities contained in the expressions.
When teaching factoring it is important to explore the rationale for factoring expressions and that the students will use the techniques of factoring in simplifying expressions such as: $\frac{x^2-1}{x-1}$
and in solving equations such as $x^2-5x-6=0$, etc.
An extension for higher-level students is to teach them the completion of the square for solving quadratic equations and analysing the quadratic function. Higher levels students might perhaps explore the use of completing the square in deriving the quadratic formula. What is meant here is that we insist that in solving quadratic equations where the quadratic has no convenient roots that we insist that they use the completion of the square to solve such equations to two decimal; places say (See attached Example 1). Why? Well, completing the square is a very useful technique in a raft of areas later on

Example 1: The Quadratic

Example 1

Solve $x^2-6x-3=0$

$$x^2-6x-3=0$$
$$x^2-6x=3$$
$$x^2-6x+9=3+9$$
$$(x-3)^2=12$$

⇨

$$(x-3)^2=12$$
$$x-3=\pm\sqrt{12}$$
$$x=3+\sqrt{12} \text{ or } x=3-\sqrt{12}$$

Use the completion of the square method to analyse the graph of the function
$f(x)=x^2-6x-3=0$

Example 2

Use the completion of the square method to derive the formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$ax^2 + bx + c = 0$$
$$ax^2 + bx = -c$$
$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$
$$x^2 + \frac{b}{a}x + \frac{b^2}{4a} = \frac{b^2}{4a} - \frac{c}{a}$$

$$\left(x - \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a}$$
$$x - \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a}}$$
$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 3

Find the roots and minimum value of the function $f(x) = x^2 - 4x + 3$. Hence plot a rough

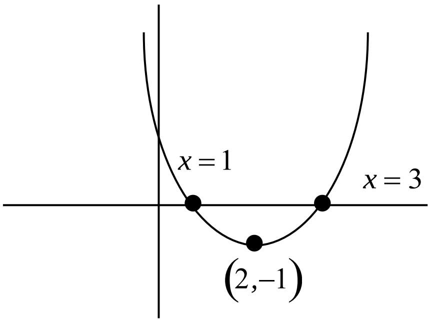
Writing the function in perfect square form:

$$f(x) = x^2 - 4x + 3 = x^2 - 4x + 4 - 1$$
$$= (x-2)^2 - 1$$
$$x^2 - 4x + 3 = 0$$
$$x^2 - 4x = -3$$
$$x^2 - 4x + 4 = -3 + 4$$
$$(x-2)^2 = 1$$
$$x - 2 = \pm 1$$
$$x = 3, x = 1$$

The roots are $x = 1, x = 3$

We know that the smallest value that $(x-2)^2$ can have is zero and that this occurs when $x = 2$

Therefore the smallest value of $f(x) = x^2 - 4x + 3$ is $0 - 1 = -1$ and this occurs at the point $(2, -1)$



- ✓ **Key Skills: Develop students' fluency and skills in relation to manipulating rational expressions**

Beginning with the rules for adding fractions as they have done in primary school bring them to expressions such as :

$\frac{1}{x-1}+\frac{1}{x+1}$, while all the time underlying the work done earlier involving patterns and the common identities. A possible extension is to introduce them to partial fractions, i.e. find A and B where

$\frac{1}{x^2-1}=\frac{A}{x-1}+\frac{B}{x+1}$ (Higher only)

- ✓ **Key Skills: Enhance students' understanding of equations. Solving equations (including simultaneous equations)**

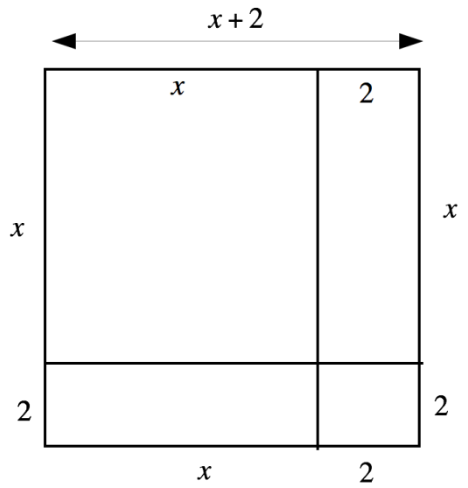
In solving equations, the distinction between an identity (which is always true $x^2-1=(x-1)(x+1)$, an expression, which is just a statement and an equation, which is true for some number of values of the variable should be drawn. Also solving equations should be intrinsically ***linked with the graph of the function and an understanding of the relationship the roots of an equation and the graph of the function***. Why has a quadratic equation as many as two roots? When does it have none? Etc. It is suggested that the students should be encouraged to draw rough sketches of quadratic functions to illustrate the solutions to quadratic equations-This has huge knock-on benefits when solving quadratic, rational and modulus inequalities later on. GeoGebra or some other software could be integrated into lesson delivery here.
It should also be possible to include real life examples for ordinary level student of how linear expressions can be created.

Example 4

A worksheet is provided with questions and discussion points arising out of the different experiments. The purpose of these worksheets is to underpin the idea that linear equations can be used to solve context-based problems.
The students will be expected to conduct surveys of the cost of activities that involve a down payment with fixed term payments and to use the techniques learned during the earlier exercises to represent and analyse the outcomes of the survey. For example, if membership of a golf club involves an initial payment of € 5,000.00 with an annual stipend of € 500.00, then the total cost (*y*) after (*x*) years is given by $y = 500x + 5000$ etc.

Similarly if a student walks towards a wall starting from 500 m away at a steady speed of $5ms^{-1}$ say, the distance from the wall after *t* seconds is given by $s = 500 - 5t$. So the students can analyse expressions provided by the teacher or can create their own by experiment or survey (contacting golf clubs, leisure centres etc). They should be encouraged to draw the graphs and answer questions from the resulting graphs.

Higher level students might look at the prefect square formula geometrically. The students will investigate the perfect-square formula and to create a geometric model to demonstrate it. For example, $(x+2)^2 = x^2 + 4x + 4$ is represented geometrically as:



- ✓ **<u>Key Skills: Enhance students' understanding of Inequalities.</u>**

Linear, quadratic etc. Insist on graphical solutions in every case

- ✓ **<u>Key Skills: Problem Solving - These should motivate the Algebraic identities and algebraic material generally and reinforce the students' understanding and appreciation of algebra</u>**

Example 5

1. The hypotenuse of a right triangle is 10 m long. One leg is two more than the smaller leg. Draw and label a diagram and write an equation using Pythagorean Theorem. Solve the equation and find the lengths of the sides of the triangle.

2. The length of a rectangle is one less than three times the width. The area is 70 cm^2. Find the length and
3. The diagonal of a square is $5\sqrt{2}$ cm. Find the length of the side of the square.

4. A 19 m wide building has a roof that rises 50 cm for every 1.5 m horizontal change. If the roof has an overhang of 1m what is the length of slant height of the roof?
5. Your aunt and uncle have decided to build a fence around their pigsty to keep the pigs from running around the farm. They were given 12 meters of free fencing from a kind neighbour with which they will construct their sty. They want to build this sty along side of the barn, so the fence only needs to cover the other 3 sides of the sty. By using only the 12 meters of fence they were given, how can you help your aunt and uncle build the largest possible sty for the pigs to live in?
6. A football is kicked into the air and follows the path $h = 16x - 2x^2$ where *x* is the time in seconds and *h* is the height in metres.
    a. What is the maximum height of the football?
    b. How long does the ball stay in the air?
    c. How high is the ball after 6 seconds?
    d. How long does it take the ball to reach a height of 15 m?
    e. Is it likely that this experiment took place on the Earth?
7. An ice cream specialty shop currently sells 240 ice cream cones per day at a price of €3.50 each. Based on results from a survey for each €0.25 decrease in price, sales will increase 60 cones per day. If the shop pays €2.00 for each ice cream cone, what price will maximize the revenue?

✓ **Key Skills: Collaborative Project to deepen students' understanding of functions and algebra**

Choose a project from the list:

- Use GeoGebra to create a teaching tool to demonstrate the properties of the different types of functions
- Use GeoGebra to create a teaching tool to demonstrate the effect of combining different functions in pairs (addition, division, multiplication, composition)
- Investigate the identities $x^2 - y^2 = x - y$ $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

and generate appropriate geometrical representations to establish their veracity

Create a set of questions on algebra suitable for (a) a higher-level class in second year and (b) an ordinary- level class in second year

2. Trigonometry

✓ **<u>Key Skills: The students will have an enhanced understanding of the trigonometric ratios</u>**

An introduction to $\sin A$, $\cos A$ and $\tan A$ as interrelated special fractions and tools to enable us to construct angles and triangles.
Higher level students might be invited to look at problems of the type:
If $\sin A = x$. Find
Tan A and $\cos A$ for the acute angle A.
Show that $Sin(A) = Cos(90^0 - A)$
Prove the identity $\sin^2 x + \cos^2 x = 1$ for *x* acute.
Use this identity to solve simple related identities and explicitly exploit this process to reinforce the algebraic techniques encountered during the earlier module on algebra.

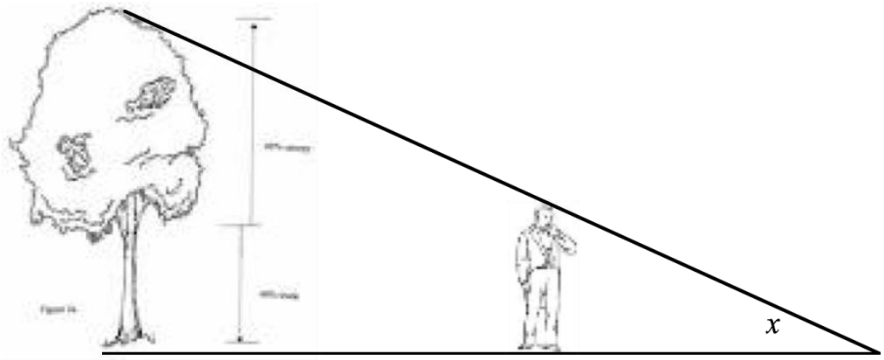
✓ **<u>Key Skills: The students will explore and develop an understanding of the application of the simple ratios to solving real life experiments.</u>**

These might be done by experiment supported by calculation. (See the following example)

Example 6

To measure the height of a tree or local tower etc.

- On a sunny day (one can dream!) bring the class out and measure the length of the shadow cast by a member of the group (who's height is known).
- Now measure the length of the shadow cast by the tower and use $\tan x$, where x is the angle of elevation and similar triangles to (two estimates can be found as a result) to measure the height of the tower.

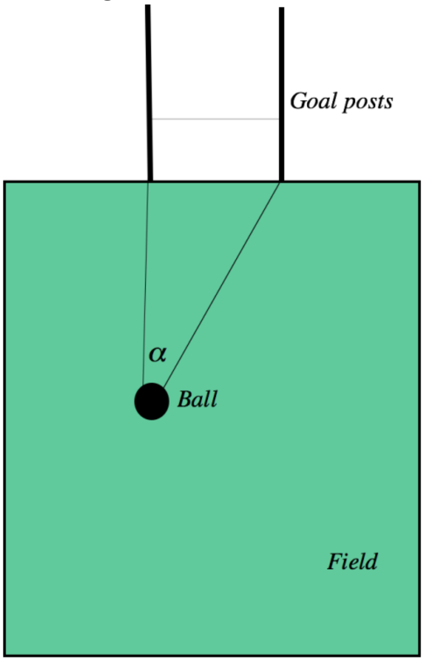


- The students can also engage in a mind experiment to see how the might measure the height of the spire or calculate the length of its shadow on a day where the angle of elevation is known.

- ✓ **<u>Key Skills: Students will revise (or be introduced to) the unit circle and become fluent in evaluating Sin A, Cos A and Tan A for angles greater than 90^0. Students will use the Sine Rule to solve a range of problems in trigonometry (Higher)</u>**

Example 7 (Higher Level)

Analysis of a rugby video and re-creation of one of the kicks undertaken by the kicker. Model the problem and determine the maximum angle the ball can be kicked in order to score.



The problem involves drawing scale diagrams to represent the position depicted in the video and using the mobile technology measure the angular reparation of the posts as measured from the position of the ball. The value of angle (α) is also determined by calculation using basic trigonometric ratios and the sine rule. The curricular areas involved in this activity include geometry, ratio and proportion, algebra and measurement. The curricular areas of statistics-a comparison of the performance of a number of kickers, surveys of the favourite sports of the students in the school etc. can also be explored. The wider curricular areas of the health implications of watching rather than participating in sport and the globalisation of sports etc. can also be explored.

✓ **Key Skills: Interactive Project- To enhance the students' familiarity with GeoGebra and to expose them to open-ended assignments in mathematics.**

The students are asked to design an applet as a teaching tool to support any one of the objectives above:

- Show the interrelationship between *Sin A, Cos A* and *Tan A*
- Investigate the secondary ratios, $Sec A$, $Cosec\, A$ and $Cotan\, A$
- Show the manner in which the ratios are linked to, and through, Pythagoras' Theorem
- Show that apart from being a ratio, $sin(x)$ can also represent a periodic function.

This should lead to a discussion on periodicity and naturally occurring periodic occurrences (tides, daylight, Alternating current and voltage). Resources available from PHET-Colorado State University would be of great assistance in exploring the links between physical phenomena and mathematics. Google PHET and select simulations on the resulting web page.

3. Proportional Reasoning

✓ **Key Skills: The students will develop an understanding of, and ability to use, proportional reasoning to solve problems and to predict the outcomes of real or 'thought' experiments.**

Concepts such as direct and inverse proportion should be explored through investigation. Areas of richness in this regard include Chemistry (the mole concept), Physics (Ohm's law etc). The inverse square law should feature as one of the most commonly occurring relationship in nature. In pursuing this topic, special 'tricks' such as cross multiplying should be avoided as they disguise the role of proportion in solving such problems.

✓ **Key Skills: Interactive Project (Small groups) -*The students will be asked to Create a learning tool for first year students* to help them master the notions of:**

- **Direct proportion**
- **Inverse Proportion**
- **Inverse square law**

For example, they might create a chart (paper or ICT-based) for fifth year science students explaining the role of proportional reasoning in their subjects
Create a series of word puzzles exploiting the confusion arising from proportional reasoning; For example 4 mice eat 4 kilos of cheese in 4 minutes, how long will 99 mice take to eat 99 kilos of cheese?
What happens to the area of a square when its perimeter is doubled? Is it different for a rectangle?
How many cm^2 are there in $1m^2$?

The games, puzzles etc should be designed with the audience in mind and perhaps a table quiz at the end of the year featuring puzzles from the various curricular strands might be held.

Example 8

Motivating Algebra and Proportional Reasoning through Problem Solving

1. Show that at most two numbers can have the same perfect square.

2. How many numbers from 1 to 100 have the figure '5' in them?

3. Two cats together catch sixty mice. If Rosie catches three mice for every two that Josie catches, how many does Josie catch?

4. A recipe, sufficient for 8, flapjacks, is shown below. Rewrite the recipe to show the mount required to make 20 flapjacks

Butter	2 Oz
Sugar	3 Oz
Rolled Oats	4 Oz

5. Our class decided to make 'silver line' to raise money for a local charity. WE invited people to put 20 cent pieces edge to edge to make a long line. The completed line was 30 m long. Roughly how many coins were in the line and how much money was raised? If the coins were rearranged to make a circle how large would the circle be? How much in total did the coins weigh?

6. How many pairs of numbers of the form $x, 2x+1$ are there in which both numbers are prime and less than 100?

7. We can write 265 as $3\left[4\right]5$, where the encasement represents a negative number so $265 = 3(100) - 4(10) + 5$. Using this notation how could (a) 356, (b)1898 and (c) 33678 be written?

8. If the numbers $\frac{1}{3}, \frac{3}{10}, 31\%, 0.03, 0.303$ are written in order (smallest first) which one will be in the middle?

9. A knitted scarf uses three balls of wool. Initially there are *b* balls of wool and *s* scarves are knitted. How many balls of wool remain?

10. I am thinking of a rule that converts the number 6 into the number 20. Which of the following could not be the rule?

    a. Add 14
    b. Take half and add 17
    c. Treble and add 2
    d. Add 4 then square
    e. Subtract 2 then multiply by 5

4. Area, Measure and Geometry

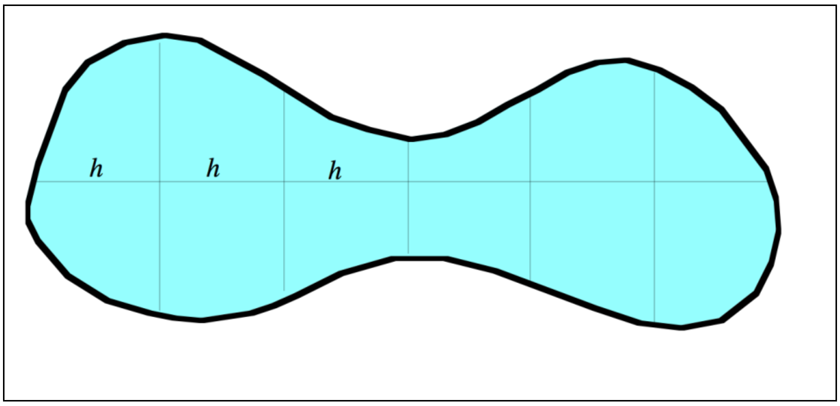
✓ **<u>Key Skills: The students will develop an appreciation of the features of the basic geometric shapes</u>**

- They can explore the properties of the pentagon, for example. Determine the angle at each vertex and explain why the angle adopts this value. How the area of such shapes is measured.
- Students will become familiar with the trapezoidal rule and the notion that the volume of an object of that doesn't taper is always found by multiplying the SA by the depth.

The students are encouraged to draw various plane figures: the different triangles, parallelogram, rhombus, rectangle, square, and various regular polygons using GeoGebra or equivalent.

<u>Example 9</u>

Measurement of the area of a flowerbed or some other irregular shape in the school grounds. The area is staked out as shown below. Scaled diagrams are drawn and a range of techniques including the trapezoidal and Simpson's rule is used to determine the area of the shape.



The activity can be extended to determination of the volume of water in a local lake. Again the surface area of the lake is established and the average depth is also found (Research). The volume of water is found by $V = SA \times Average\ Depth$. The curricular areas involved in this activity include geometry, ratio and proportion, algebra and measurement. The wider curricular areas of environment, chemistry and public policy relating to the provision and maintenance of public facilities can also be explored.

- ✓ **Key Skills: Problem Solving- The students' collaboration, communication, literacy and problem-solving skills will be developed.**

Determine the area of your local county and calculate the error in your calculation with reference to the data on file at the local county council offices. Any method, including the use of squared paper, the trapezoidal or Simpson's rule is perfectly acceptable. Additional tasks such as calculating the population density of the county and so on can then be pursued

- ✓ **Key Skills: Interactive Project- Further develop the students' ICT and GeoGebra skills**

The students create works of art using the transformational geometry functions built into the software.