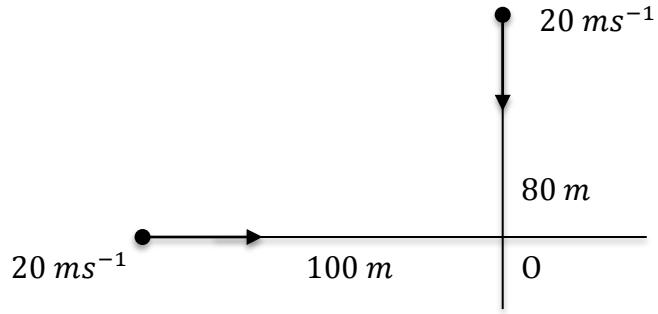
APPLICATIONS OF CALCULUS

1 The Point O is the intersection of two roads that cross at right angles as shown. One car travels towards O from the north at $20ms^{-1}$ while the second travels due east towards O also at $20ms^{-1}$.



(a) Show that after t seconds their distance apart, d, is given by

$$d = \sqrt{(100-20t)^2 + (80-20t)^2}$$

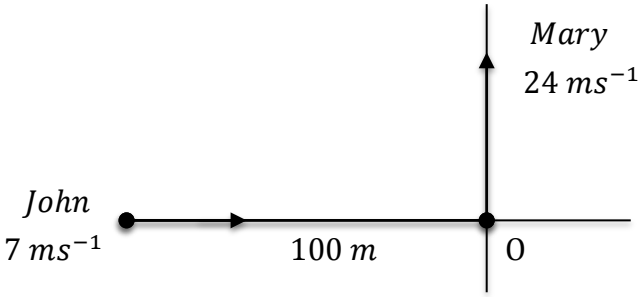
(b) Show that this simplifies to

$$d^2 = 400\left[(5-t)^2 + (4-t)^2\right]$$

(c) Show, using calculus that the minimum distance between the two cars is $10\sqrt{2}\ m$.

(d) Now show, without using calculus, that the minimum distance between the two cars is $10\sqrt{2}\ m$

2 Two straight roads cross at right angles at O. John is running along one of the roads towards O at 7 ms^{-1} . Mary is cycling along the other road at 24 ms^{-1} . When John is 100 m from O, Mary is at O.



(i) Find an expression for the distance between John and Mary.

(ii) Hence find the shortest distance between John and Mary at any time t.

1987 QUESTION 2b: AN ALTERNATIVE TO USING $V_A = V_B$

A car starts from a point O with an initial speed of 8 ms^{-1} and then travels with a uniform acceleration of 4 ms^{-2}. Two seconds later a second car Q starts with an initial velocity of 30 ms^{-1} and then moves with a uniform acceleration of 3 ms^{-2}.

Show that after passing P, Q will never be ahead by more than 74m.

(i) Find an expression for the distance travelled by car P at time t seconds

(ii) Find an expression for the distance travelled by car Q at time t seconds

(iii) Find an expression for the distance between the two cars

(iv) Find the time when distance between them is minimum

(v) Find minimum distance between them

1992 QUESTION 1b

Two particles P and Q are moving in the same direction along parallel straight lines. Their accelerations are 5 m/s^2 and 4 m/s^2, respectively.
At a certain instant P has a velocity 1 m/s and Q is 25.5 m behind P moving with velocity 11 m/s.

(i) Prove that Q will overtake P and that P will in turn overtake Q.
(ii) When Q is in front of P find the greatest distance between the particles.

Find expression for distance travelled by P

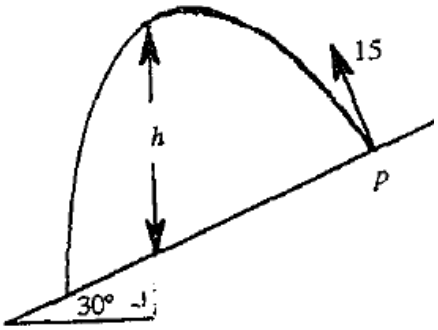
Find expression for distance travelled by Q

Find an expression for the distance, D, between the particles when Q is in front of P

Find the maximum distance between them

1997 QUESTION 3b

A particle is projected from a point p with initial speed 15m/s, down a plane inclined at an angle of 30^0 to the horizontal. The direction of projection is at right angles to the inclined plane.



(The plane of projection is vertical and contains the line of greatest slope).

Find

(i) the perpendicular height of the particle above the plane after t seconds and hence, or otherwise, show that the vertical height h of the particle above the plane after t seconds is $10\sqrt{3}\,t - 4.9t^2$

(ii) **the greatest vertical height it attains above the plane (i.e. the maximum value of h) correct to two places of decimals.**